CANTILEVER :-

A Cantilever is a uniform beam fixed horizontally at one end and loaded at the other end.
We consider, the cantilever fixed at O and loaded at the free end by a concentrated weight W.
Let P(x,y) any point on the beam and 'l' be the length of the beam. The applied bending moment due to the load 'W' about the point 'P' which is at a distance x from the fixed end $= W(l-x)$.

Hence, the bending moment equation is

$$\frac{YAK^2}{R} = W(l-x)$$

The L.H.S represents the internal bending moment

$$\text{or, } YAK^2\frac{d^2y}{dx^2} = W(l-x)$$

{when the bending is small the curvature $\frac{1}{R}$ at P may be taken to be equal to $\frac{d^2y}{dx^2}$}

integrating we get

$$YAK^2\frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right) + C_1$$

Now, $\frac{dy}{dx} = 0$, when $x = 0$

because at the ~~end~~ fixed end the bar remains horizontal.

$$C_1 = 0$$

then, $YAK^2\frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right)$

~~on furthe~~

on further integration, we have.

$$YAK^2y = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$$

Again, $y=0$, when $x=0$, therefore, $C_2 = 0$.

$$\therefore YAK^2y = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$$

This gives the expression for the depression of the beam at any point.

Now, at the point of loading, $x=l$ and δ be the depression of the beam.

then,

$$YAK^2\delta = W\left(\frac{l^3}{2} - \frac{l^3}{6}\right) = \frac{Wl^3}{3}$$

$$\text{or, } \delta = \frac{Wl^3}{3YAK^2} \quad \text{———} \quad (1)$$

If the beam be of rectangular cross-section of breadth a and thickness b, we have

$$AK^2 = \frac{ab^2}{12}$$

$$\therefore \delta = \frac{Wl^3}{3Y \cdot \frac{ab^2}{12}} = \frac{4Wl^3}{Yab^3} \quad \text{———} \quad (2)$$

Now, due to the load W each element of the beam experiences a shearing stress of magnitude $\frac{W}{ab}$.

As a result of this stress there will be depression at each point of the beam.

If 'l' be the length of the beam and δ_1 be the depression at the end due to shear, we get

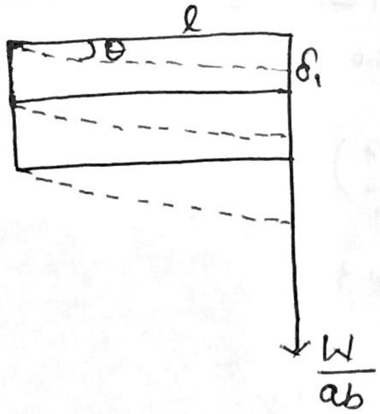
$$\frac{\delta_1}{l} = \theta \qquad \therefore \eta = \frac{\frac{W}{ab}}{\frac{\delta_1}{l}} = \frac{Wl}{\delta_1 ab} \qquad \text{—— (3)}$$

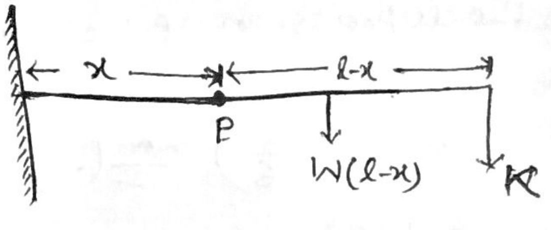
η being the modulus of rigidity and θ the shearing strain,

$$\delta_1 = \frac{Wl}{\eta ab}$$

$$\therefore \frac{\delta_1}{\delta} = \frac{\frac{Wl}{\eta ab}}{\frac{4Wl^3}{Yab^3}} = \frac{Y}{4\eta}\left(\frac{b}{l}\right)^3$$

Now, b is Negligibly small in Comparision With l.
Hence δ_1 is insignificant in Comparison with δ.
In the above deduction, we have ignored the weight of the Cantilever.
 let, the beam has weight K per unit length and Carries a Concentrated load W at its free end.





Then, bending moment equation at P is

$$YAK^2\frac{d^2y}{dx^2} = W(l-x) + K\frac{(l-x)\cdot(l-x)}{2}$$

$$= W(l-x) + \frac{K}{2}(l-x)^2$$

$$\Rightarrow YAK^2\frac{d^2y}{dx^2} = W(l-x) + \frac{K}{2}(l^2+x^2-2lx)$$

Integrating, we get

$$YAK^2\frac{d^2y}{dx} = W\left(lx-\frac{x^2}{2}\right) + \frac{K}{2}\left(l^2x+\frac{x^3}{3}-lx^2\right) + C_1$$

on Further, integration, we get

$$YAK^2y = W\left(\frac{lx^2}{2}-\frac{x^3}{6}\right) + \frac{K}{2}\left(\frac{l^2x^2}{2}+\frac{x^4}{12}-\frac{lx^3}{3}\right) + C_1x + C_2$$

Now, at $x=0$, $\frac{dy}{dx}=0$, and $y=0$

$$\therefore C_1 = C_2 = 0$$

$$\therefore YAK^2y = W\left(\frac{lx^2}{2}-\frac{x^3}{6}\right) + \frac{K}{2}\left(\frac{l^2x^2}{2}+\frac{x^4}{12}-\frac{lx^3}{3}\right)$$

If δ be the depression of the beam at the free end where the load is applied, we have.

$$YAK^2\delta = W\left(\frac{l^3}{2}-\frac{l^3}{6}\right) + \frac{K}{2}\left(\frac{l^4}{2}+\frac{l^4}{12}-\frac{l^4}{3}\right)$$

$$= \frac{Wl^3}{3} + \frac{Kl}{2}\cdot\frac{l^3}{4} = \frac{Wl^3}{3} + \frac{W_1l^3}{8}$$

Where $W_1 = K$ is the weight of the beam.

$$\therefore \boxed{\delta = \frac{l^3}{3YAK^2}\left(W+\frac{3}{8}W_1\right)}$$

Teacher's Signature

$= X =$